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# GENERALIZED REVERSE DERIVATIONS ON PRIME RINGS

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## ABSTRACT

In this paper we present Some results concerning to reverse derivations on prime ring $R$ to the right generalized reverse derivations associated with a derivation $d$o f $R$ and anon-zero left ideal $U$ of $R$ which is semi prime asaring. We prove that if $f$ is aright generalized reverse derivation of a prime ring $R$, $U$ is an on-commutative left ideal of $R$ and$[x,f(x)]=0$or$[f(x),f(y)]=[x, y]$ or $f(U)\subseteq Z$ then there exists Martindal erring of quotientsii e.,$q\in Q_r(R_c)$such that$f(x)=qx$, for all $x \in R$

**KEYWORDS:** Prime Ring, Semi Prime Ring, Derivation, Generalized Derivation, Reverse Derivation, Generalized Reverse Derivation, Martindale Ring

## INTRODUCTION

Bresar [1] defined generalized derivation of rings. Hvala [5] studied the properties of generalized derivation sin prime rings. I.N.Herstein [4], Bresarand Vukman [2] have introduced the concept of reverse derivation so f prime ring sand studied the notation of reverse derivation sand some properties of reverse derivations. Golbasi [3] extended some well known results concerning oderivations of prime ring s to the right generalized derivations and an on-zeroleftidealofaprimeringwhichissemiprimeasaring.K.SuvarnaandD.S.Irfana[6]studiedsomeresultsconcerningtoderivationso f prime ring s to generalized derivations and an on-zero right ideal of a prime ring which is semi prime as a ring.

An additive map $d$ from a ring $R$ to $R$ is called a derivation if $d(xy)=d(x)y+xd(y)$for all $x,y$ in $R$. An additive map $d$ from a ring $R$ to $R$ is called are verse derivation if $d(xy)=d(y)x+yd(x)$for all$x,y$ in$R$.An additive mapping $f:R\to R$ is said to be a right generalized derivation if there exists aderivation $d$ from $R$ to $R$ such that $f(xy)=f(x)y+xd(y)$, for all $x$, $y$ in $R$. An additive mapping $f$: $R\to R$ is said to be a left generalized derivation if there exist s aderivation $d$ from R to $R$ such that f $(xy)=d(x)y+xf(y)$, for all $x$, $y$ in $R$. An additive mapping $f$: $R\to R$ is said to be a generalized derivation if it is both right and left generalized derivation.

We know that an additive mapping $f$: $R\to R$ is a right generalized reverse derivation if there exists a derivation $d$ from $R$ to $R$ such that $f(xy)=f(y)x+yd(x)$, for all $x,y$ in $R$ and $f$ is a left generalized reverse derivation if there exists a derivation $d$ from $R$ to $R$ such that $f(xy)=d(y)x+yf(x)$,for all $x$, $y$ in $R$. Finally, f is a generalized reverse derivation of $R$ associated with $d$ if it is both right and left generalized reverse derivation of $R$. Through out this section, $R$ will be aprimering of char.$\neq 2$, $U$ an on-zeroleft ideal of $R$ which is semi primeasaring, $Z$ the center of $R$,$Q_r(R_C)$the Martindale ring of quotients, $C$ the extended centroid and $R_C=R_C$ the central closure

First we prove the following Lemmas:

**Lemma1:** Let R be prime ring and $U$ an on-zero left ideal of $R$ which is semi prime as a ring.If $ua = 0 (au = 0)$ for all $a \in R$, then $a = 0$.

**Proof:** Since $U \neq \{0\}$,there exist an element $u \in U$ such that $u \neq 0$.Considerthat $u\ Ra \subset Ua = \{0\}$.Since $u \neq 0$ and $R$ is a prime ring, wehavethat $a = 0$. ■

**Lemma2:**Let $f: R \to R_C$ be an additive map satisfying $f(x)y = xf(y)$, for all $x, y \in R$.

Then there exists $q \in Q_r(R_C)$ such that $f(x) = qx$, for all $x \in R$.

**Proof:**we extend $\bar{f}$ from $R$ to $R_C$ such that $\bar{f}(\sum \lambda_i x_i) = \sum \lambda_i f(\bar{x_i})$ for all $x_i \in R$ and $\bar{\lambda_i} \in C$. Now to show that $\bar{f}$ is well defined, it is sufficient to prove that $\sum \lambda_i x_i = 0$ implies $\sum \lambda_i f(x_i) = 0$.Let $U$ be a non-zero ideal in $R$ such that $U\lambda_i \subseteq R$ for every $i$. Let $a \in U$ and we note that factors in the sum $\sum (a\lambda_i)x_i$ lie in $R$. There fore we have $f(\sum \lambda_i x_i) = 0$ implies $a(\sum \lambda_i f(x_i)) = 0$.Since this is true for all $a \in U$, we have $\sum \lambda_i f(x_i) = 0$.By direct computation, we have $f(xy) = xf(y)$. This proves that $f: R_C \to R_C$ is a right $R_C$-module map. Hence there exists $q \in Q_r(R_C)$ such that $f(x) = qx$, for all $x \in R_C$.Since $f$ is an extension of $f$, this proves the lemma.

**Lemma3:** Let be a prime ring and $U$ an on-zeroleftideal of $R$ which is semi prime as a ring .If $d$ is a reverse derivation of $R$ such that $d(U) = 0$, then $d = 0$.

**Proof:**Forall $x \in U, r \in R$,weget,

$0 = d(rx) = d(r)x$,

And so, $d(R)U = 0$

ByLemma:1, we obtain that $d = 0$. ■

**Theorem1:**Let $R$ be a primering, $U$ an on-zero left ideal of $R$ which is semi prime as a ring and $f$ a Right generalized reverse derivation of $R$. If $U$ is non-commutative and $f([x, y]) = 0$, forall $x, y$ in $U$,thenthereexists $q \in Q_r(R_C)$ such that $f(x) = q\,x$, for all $x \in R$.

**Proof:** Given that $f([x, y]) = 0$, forall $x, y$ in $U$.

We replace $y$ by $xy$ in $f([x, y]) = 0$,then,

$\Rightarrow f([x, xy]) = 0$

$\Rightarrow f(x[x, y] + [x, x]y) = 0$

$\Rightarrow f(x[x, y]) = 0$

$\Rightarrow f([x, y])x + [x, y]d(x) = 0$

$\Rightarrow [x, y]d(x) = 0$

Replace $y$ by $ry$ in the above equation, we get,

$\Rightarrow [x, ry]d(x) = 0$

$\Rightarrow (r[x, y] + [x, r]y)d(x) = 0$

$\Rightarrow r[x, y]d(x) + [x, r]yd(x) = 0$

Since the first summand is zero, it is clear that

$[x, r]yd(x) = 0$, for all $x, y \in U, r \in R$.

By replacing $sy, s \in R$ in place of $y$ in the above equation, we get,

$\Rightarrow [x, r]syd(x) = 0$

Since $R$ is a prime ring, we have $Ud(x) = 0$ (OR) $[x, r] = 0$, for all $x \in U, r \in R$.

By Lemma:1, we get either $d(x) = 0$ or $x \in Z$, for all $x \in U$.

Let $A = \{x \in U / d(x) = 0\}$ and $B = \{x \in U / x \in Z\}$.

Then $A$ and $B$ are two additive sub groups of $(U, +)$ such that $U = A \cup B$.

However, a group cannot be the union of proper sub groups. Hence either $U = A$ or $U = B$. If $U = B$, then $U \subset Z$, and so, $U$ is commutative, which contradicts the hypothesis. So, we must have $d(x) = 0$, for all $x \in U$. By Lemma:3, we get, $d = 0$. Hence there exists $q \in Q_r(R_C)$ such that $f(x) = qx$, for all $x \in R$, by Lemma: 2. ∎

**Theorem 2:** Let $R$ be a primering with char $\neq 2$, $U$ an on-zero left ideal of $R$ which is semi prime as a ring and $f$ a Right generalized reverse derivation of $R$. If $U$ is non- commutative and $[x, f(x)] = 0$, then there exist s $q \in Q_r(R_C)$ such that $f(x) = qx$, for all $x \in R$.

**Proof:** Given that $[x, f(x)] = 0$

By linearizing $[x, f(x)] = 0$, we get,

$\Rightarrow y[x, d(x)] + [x, y]d(x) = 0$, for all $x, y \in R$ … .. (1)

We replace $y$ by $z\,y$ in (1) and using equ.(1), we obtain that,

$\Rightarrow zy[x, d(x)] + [x, zy]d(x) = 0$

$\Rightarrow zy[x, d(x)] + (z[x, y] + [x, z]y)d(x) = 0$

$\Rightarrow -z[x, y]d(x) + z[x, y]d(x) + [x, z]yd(x) = 0$

$\Rightarrow [x, z]yd(x) = 0$, for all $x, y, z \in U$ ….. (2)

By replacing $y$ by $ry$, $r \in R$ in equ.(2) and since $R$ is prime, we get, $[x, z] = 0$ or $U\,d(x) = 0$, for all $x, z \in R$. By Lemma: 1, we have either r$[x, z] = 0$ or $(x) = 0$, for all $x \in U$. By a standard argument, one of these must be held for all $x \in U$. The first result cannot hold, since $U$ is non-commutative, so, the second possibility give s $d(U) = 0$ and hence $d = 0$. Therefore, the proof is completed by using

**Lemma 2**

**Theorem 3:** Let $R$ be a primering with char $. \neq 2, U$ an on-zero left ideal of $R$ which is semi prime as a ring and $f$ a Right generalized reverse derivation of $R$. If $U$ is non- commutative, $d(z) \neq 0$ and $[f(x), f(y)] = [x, y]$, for all $x, y \in U$, then there exists $q \in Q_r(R_C)$ such that $f(x) = qx$, for all $x \in R$.

**Proof:** From the hypothesis we have, $[f(x), f(y)] = [x, y]$, for all $x, y \in U$ ….. (3)

By taking $xy$ instead of $y$ in equ.(3), weget,

$$\Rightarrow [x, xy] = [f(x), f(xy)]$$

$$\Rightarrow x[x, y] + [x, x]y = [f(x), f(y)x + yd(x)]$$

$$\Rightarrow x[x, y] = f(x)(f(y)x + yd(x)) - (f(y)x + yd(x))f(x)$$

$$\Rightarrow x[x, y] = f(x)f(y)x + f(x)yd(x) - f(y)xf(x) - yd(x)f(x)$$

$$\Rightarrow x[x, y] = f(x)f(y)x - f(y)xf(x) + f(x)yd(x) - yd(x)f(x)$$

$$\Rightarrow x[x, y] = [f(x), f(y)x] + [f(x), yd(x)]$$

$$\Rightarrow x[x, y] = f(y)[f(x), x] + [f(x), f(y)]x + y[f(x), d(x)] + [f(x), y]d(x)$$

Byusing equ.(3),we get,

$$\Rightarrow x[x, y] = f(y)[f(x), x] + [x, y]x + y[f(x), d(x)] + [f(x), y]d(x),$$

For all $x, y \in U$ ….. (4)

We replace $y$ by $cy = yc$, where $c \in Z$ and using equ.(4), we obtain,

$$\Rightarrow x[x, cy] = f(cy)[f(x), x] + [x, cy]x + cy[f(x), d(x)] + [f(x), cy]d(x)$$

$$\Rightarrow x(c[x, y] + [x, c]y) = (f(y)c + yd(c))[f(x), x] + (c[x, y] + [x, c]y)x + cy[f(x), d(x)] + (c[f(x), y] + [f(x), c]y)d(x)$$

$$\Rightarrow xc[x, y] + x[x, c]y = f(y)c[f(x), x] + yd(c)[f(x), x] + c[x, y]x + [x, c]yx + cy[f(x), d(x)] + c[f(x), y]d(x) + [f(x), c]y d(x)$$

Since c is commutative, we have,

$$\Rightarrow cx[x, y] + x[x, c]y = cf(y)[f(x), x] + yd(c)[f(x), x] + c[x, y]x + [x, c]yx + cy[f(x), d(x)] + c[f(x), y]d(x) + [f(x), c]yd(x)$$

$$\Rightarrow cx[x, y] + x[x, c]y = cf(y)[f(x), x] + c[x, y]x + cy[f(x), d(x)] + c[f(x), y]d(x) + [x, c]yx + [f(x), c]yd(x) + yd(c)[f(x), x]$$

$$\Rightarrow cx[x, y] + x[x, c]y = c(f(y)[f(x), x] + [x, y]x + y[f(x), d(x)] + [f(x), y]d(x)) + [x, c]yx + [f(x), c]yd(x) + yd(c)[f(x), x]$$

From equ.(4), we have,

$$\Rightarrow cx[x, y] + x[x, c]y = cx[x, y] + [x, c]yx + [f(x), c]yd(x) + yd(c)[f(x), x]$$

$\Rightarrow x[x, c]y = [x, c]yx + [f(x), c]yd(x) + yd(c)[f(x), x]$

$\Rightarrow x(xc - cx)y = (xc - cx)yx + [f(x), c]yd(x) + yd(c)[f(x), x]$

$\Rightarrow x(xc - xc)y = (xc - xc)yx + [f(x), c]yd(x) + yd(c)[f(x), x]$

$\Rightarrow 0 = 0 + [f(x), c]yd(x) + yd(c)[f(x), x]$

$\Rightarrow 0 = [f(x), c]yd(x) + yd(c)[f(x), x]$

Since $d(z) \neq 0 \Rightarrow 0 \neq d(c) \in Z \Rightarrow d(x) = 0$, for all $x \in U$

$\Rightarrow yd(c)[f(x), x] = 0$

Since $d(z) \neq 0 \Rightarrow 0 \neq d(c) \in Z$ and $U$ is a non-zero left ideal of $R$, wehave,

$\Rightarrow [f(x), x] = 0$, for al l$x \in U$.

The proof is now completed byusingTheorem:2. ■

**Theorem4:**Let$R$beaprimeringwithchar$\neq 2$,$U$ an on-zero left ideal of $R$ which is semi prime a saring and $f$ a Right generalized reverse derivation of a ring $R$. If $U$ is non-commutative and$f(U) \subseteq Z$, then there exists $q \in Q_r(R_C)$such that$f(x) = qx$,forall$x \in R$.

**Proof:** Forall$x \in R$, we have,$[f(yr), y] = 0$

So, $[f(yx), y] = 0$, for all$x, y \in U$

$\Rightarrow [f(x)y + xd(y), y] = 0$

$\Rightarrow x[d(y), y] + [x, y]d(y) = 0$

Byexpandingthis equation, weget,

$\Rightarrow x(d(y)y - yd(y)) + (xy - yx)d(y) = 0$

$\Rightarrow xd(y)y - xyd(y) + xyd(y) - yxd(y) = 0$

$\Rightarrow xd(y)y - yxd(y) = 0$

$\Rightarrow xd(y)y = yxd(y)$,forall$x, y \in U$ ….. (5)

We replace $x$ by $xz$ in equ. (5), we get,

$\Rightarrow xzd(y)y = yxzd(y)$

By using equ.(5)in the above equation, we get,

$\Rightarrow yxzd(y) = x(zd(y)y)$

$\Rightarrow yxzd(y) = xyzd(y)$

$\Rightarrow xyzd(y) - yxzd(y) = 0$

$\Rightarrow (xy - yx)zd(y) = 0$

$\Rightarrow [x,y]zd(y) = 0, \text{for all } x,y,z \in U$

By taking $zr$ instead of $z$ in the above equation and using the fact that $R$ is prime, we conclude that $d(y) = 0$ or $[x,y] = 0$, for all $x,y \in U$. By the standard argument, we have either $d = 0$ or $U$ is commutative. Since, $U$ is non-commutative, the proof is completed by using Lemma:2. ■

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